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Chaos in nodal points and streamlines in ballistic electron transport through quantum dots

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Abstract. We trace signatures of quantum chaos in the distribution of nodal points and streamlines for coherent electron transport through different types of quantum dots (chaotical and regular). We have calculated normalized distribution functions for the nearest distances between nodal points and found that this distribution may be used as a new signature of quantum chaos for electron transport in open systems. All irregular billiards shows the same characteristic distribution function. These signatures of quantum chaos are well reproduced using well-known approaches of chaotic wave functions with the same characteristic distribution function. We have also investigated the quantum flows, and have found some remarkable properties of them.

Introduction

The field of quantum chaos has received much attention, due to the increase of the investigations of low-dimensional systems. The nature of quantum chaos in a specific system is traditionally inferred from its classical counterpart. Hence one may ask if quantum chaos is to be understood solely as a phenomenon that emerges in the classical limit, or if there are some intrinsically quantum phenomena, which can contribute to irregular behavior in the quantum domain. In the present work this problem is discussed in relation with ballistic quantum transport through regular and irregular electron billiards.

The eigenstates in closed irregular billiards have revealed the characteristic complex patterns of nodal lines [1]. Here we investigate the evolution of these patterns when opening the billiard and introducing a current through it. In order to clarify how the pertrubing leads reduce the symmetry and how a regular billiard may eventually turn into a chaotic one, we follow the evolution of the patterns with increasing energy.

For such an open system the wave function ψ is now a scattering state with both real and imaginary parts, each of which gives rise to separate sets of nodal lines ($Re[\psi] = 0$ or $Im[\psi] = 0$). Nodal points, i.e., the points at which these two sets of nodal lines intersect because $Re[\psi] = Im[\psi] = 0$, and their spatial distribution will play a crucual role for the characteristics of the flow in the system. The vicinity of a nodal point constitutes a forbidden area for quantum streamlines (Bohm trajectories) contributing to the net transport from source to drain [2, 3]. In our case the most important property of the nodal points of ψ is a formation of quantum vortices in the current probability flow which gives rise to the phenomenon that the quantum streamlines passing from source to drain can not skirt around the nodal points.

1. Theoretical model and calculation

For this study we use model of quantum billiard with two attached semi-infinite leads. Electrons are confined by hard wall boundaries. The interior potential is set equal to zero. We believe that principal results are not sensitive to the particular choice of boundaries. Using the dimensionless variables $x \to x/d$, $y \to y/d$ and the energy $\epsilon = 2m^* d^2 E/\hbar^2$, where d is the width of the leads, we map the Schrödinger equation for electron of a mass m^* onto a square lattice labeled (k, l) and with cell size a_0.

$$\left[4 - \left(\frac{d}{a_0}\right)^2 \epsilon\right] \psi_{k,l} - \psi_{k+1,l} - \psi_{k-1,l} - \psi_{k,l+1} - \psi_{k,l-1} = 0. \tag{1}$$

Typical grid sizes are between 200×400 and 600×1200 for rectangular stadium. Number of open transport channels was selected in a range form 1 to 20.

For introducing streamlines we use alternative interpretation of quantum mechanics. Writing the wave function in terms of a norm and a phase

$$\psi = \sqrt{\rho} \exp\left(iS/\hbar\right) \tag{2}$$

the time independent Schrödinger equation can be decomposed as follows [2, 3]

$$\begin{aligned} E = \tfrac{1}{2} m v^2 + V + V_{QM} \\ \nabla \rho \mathbf{v} = 0\,, \end{aligned} \tag{3}$$

where

$$\mathbf{v} = \nabla S/m. \tag{4}$$

Streamlines (Bohm trajectories) depends on the solution of time-dependent equations

$$\dot{x} = v_x, \dot{y} = v_y. \tag{5}$$

The nodal points were obtained as intersection of nodal lines of real and imaginary parts of wave function (lines where these parts change a sign). We propose that an appropriate signature of quantum chaos in open cavities may be formulated in the following way. The distribution of distances for the nearest neighbours of the nodal points are expected to be distinctly different for nominally regular and irregular billiards. The distribution of the distances for nearest neighbours was found in the following way. For i-th nodal point (x_i, y_i) the distance to the nearest neighbour r_i was evaluated. This was done for each channel for a given energy of incoming electron. Finally histograms for r_i was averaged over different numbers (51, 101 or more) of energy values in narrow energy interval, with a few conductance fluctuations. Then the distribution was normalized. We have also used a similar procedure for averaging over the positions of incoming leads.

In order to check our introduced signature of quantum chaos we have calculated the same distributions for a complex combination states in the nominally closed billiards (6) and also for Berry-type wave functions (7).

$$\psi(x, y) = \textstyle\sum_{mn} a_{mn} \psi_{mn}(x, y) \tag{6}$$

$$\psi(x, y) = \textstyle\sum_j a_j \exp(i\mathbf{k}_j \cdot \mathbf{r} + \phi_j), \tag{7}$$

where $\psi_{mn}(x, y)$ are discrete eigenstates of closed cavity, a_{mn}, a_j, ϕ_j are uniformly distributed coefficients and $\mathbf{k}_j$ are wave vectors of a given energy shell.

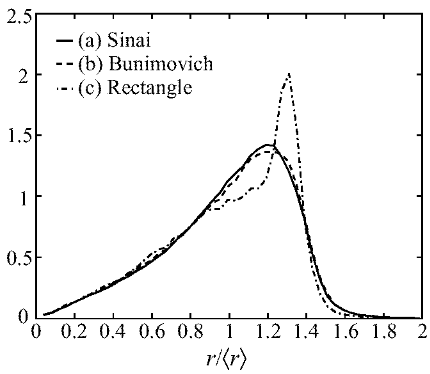

Fig. 1. Distributions of nearest distances for an electron transport through Sinai billiard $\epsilon = 50$ (a), Bunimovich stadium $\epsilon = 79$ (b), and rectangle with $\epsilon = 51$ (c), averaging by energy, size of grid is 840×400.

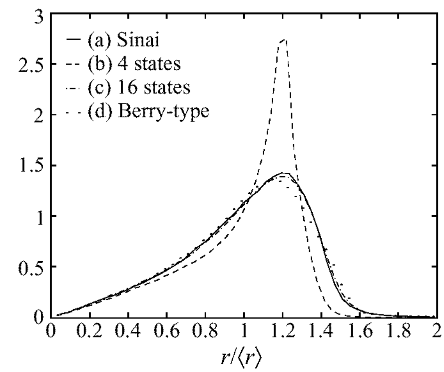

Fig. 2. Distributions for Sinai billiard (a), 4 mixed states in rectangular billiard (b), 12 mixed states (c) and Berry-type wave function (d).

2. Results and discussion

We have calculated normalized distribution functions for the nearest distances between nodal points. Our typical statistic is a few millions distances between nodal points. We have found that this distribution has a characteristic form for all types of open billiards except for rectangular ones (Fig. 1). We suggest that this form is universal one and may be used as a new signature of quantum chaos for an electron transport in the open systems. The universal distribution function is shown to be insensitive to the way of averaging (over positions of leads or over a narrow energy interval with a few conductance fluctuations). An integrable rectangular billiard yields a nonuniversal distribution for the nearest neighbour separations with a central peak corresponding to partial order of the nodal points (Fig. 1). The distributions for mixed states in the closed rectangle and for Berry-type wave function shown in Fig. 2 in comparision with our universal form for irregular cavities. Obviously, that distributions for this well-known approaches of chaotic wave functions are closed to our universal form. It confirm that our universal form can be used as the new signature of quantum chaos for an electron transport through quantum dots. Comparison of the distributions for the closed rectangle with different number of states and the distributions for regular and irregular dots let us to assert following statement. The difference between the distributions for different cavities are related to the number of eigenstates “effectively involved” into the electron transport because of symmetry.

Examples of streamlines are given in Fig. 3 and Fig. 4. We have found an effect of “channeling” of streamlines in the case of a regular cavity. This effect can be explained by partial order of nodal points. But in the same time patterns of quantum flows for irregular billiards are very complex and disordered.

Acknowledgments

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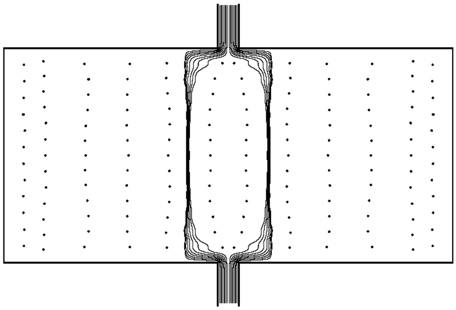

Fig. 3. Nodal lines and vortices positions for a rectangle in the tunneling regime at resonance energy $\epsilon \sim 19.2727$. The dimensions of the rectangle are $10d$ times $21d$ where d is the width of the channel. The tunneling situation is achieved by introducing appropriate barriers at the entrance and exit leads. The particle is injected through the upper lead.

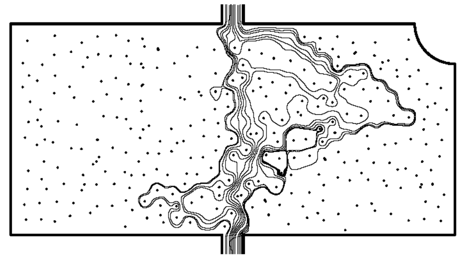

Fig. 4. Streamlines and positions of vor1tices for a Sinai billiard at an energy 20.79 (one open channel). The radius of the semicircular region is $2d$ where d is the width of the leads.

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